Non supersymmetric strings on AdS_3

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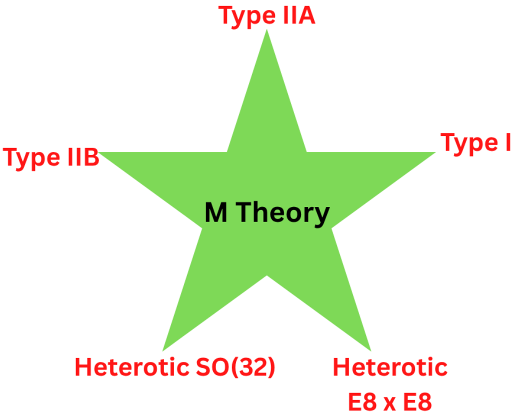
Super string theories

- 5 superstring theories (without tachyons)



Tachyons

- Tachyons are particles with negative mass squared

$$m^2_{\text{tachyon}} < 0$$

- These particles signal instability of the vacuum

- Most non supersymmetric string theories have tachyons

Non supersymmetric strings

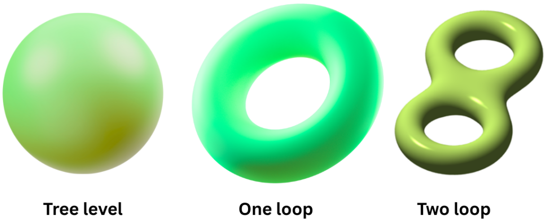
- Three 10 dimensional string theories don't have tachyons

  (i) Heterotic $O(16) \times O(16)$ theory

  (ii) Sugimoto $USp(32)$ model

  (iii) Sagnotti $U(32)$ model (also called $0'B$ model)

- We can have one loop corrections to the string theory action that can give rise to a cosmological constant



- In superstring theories, the loops corrections to the cosmological constant vanish

Uplift to deSitter

- Loop corrections are present in non supersymmetric string theories
- Can you uplift *AdS* spacetime to deSitter spacetime by these loop corrections?

Stability

- Are non-supersymmetric backgrounds stable? or at least metastable?
  - Perturbative stability

$$
\begin{cases}
m^2 \geq 0 \text{ (Minkowski)} \\
\begin{cases}
m^2_{\text{scalars}} \geq \underbrace{-L^{-2}\frac{d^2}{4}}_{\text{scalar BF bound}} \\
m^2_{\text{p-form}} \geq \underbrace{-L^{-2}\frac{(d-2p)^2}{4}}_{\text{p-form BF bound}}
\end{cases}
\text{(AdS}_{d+1}\text{ spacetime)}
\end{cases}
$$

  - Non-perturbative stability: Stability under bubbles of true vacuum engulfing the false vacuum [Coleman;1977] [Coleman & Callan ;1977] [Coleman & deLuccia; 1979]

A No-go theorem

- A Maldacena-Nunez style no-go theorem exists [Basile & Lanza; 2020] for compactifying the following action

$$S = \frac{1}{16\pi G_D}\int d^D x\sqrt{-g}\left(R - \frac{4}{D-2}(\partial\phi)^2 - Te^{\gamma\phi} - \frac{e^{\alpha\phi}}{2(p+2)!}|H_{p+2}|^2\right)$$

on a closed manifold Y with $\dim(Y) = q \geq p+2$ where H_{p+2} threads a $p+2$ cycle in Y. deSitter and Minkowski vacuua are forbidden if

$$\frac{\alpha}{\gamma} + (p+1) > 0$$

- It doesn't apply directly if we have electric and magnetic fluxes.

- $AdS_3 \times S^3 \times S^3$ vacuua have been studied for $O(16) \times O(16)$ heterotic theory on a string scale S^1 [Baykara, Robbins, Sethi; 2022],[Fraiman, Grana, Parra de Freitas, Sethi; 2023]
  - Three H_3 fluxes: n_1 on AdS_3, n_5 on S_3 and $\hat{n}_5$ on $\hat{S}_3$
  - The uplift to deSitter doesn't happen
  - Perturbative stability of scalars in AdS_3 is checked (scalars above the BF bound)
  - Non-perturbative stability still not checked

- We will try to answer the same questions for $AdS_3 \times S^3$ vacuua for $O(16) \times O(16)$ on a string scale T^4 (i.e. uplift to deSitter and perturbative stability)
  - Rich T_4 moduli space
  - Some results can be calculated analytically (in contrast to $AdS_3 \times S^3 \times \hat{S}_3 \times S^1$)
  - Results may be similar to $AdS_3 \times S^3 \times K3$ case

Tree level analysis

The $O(16)\times O(16)$ heterotic theory has the following action

$$S=\frac{1}{2\kappa_{10}^2}\int d^{10}x\, e^{-2\phi}\sqrt{-g}\left(R+4\left(\partial\phi\right)^2-\frac{1}{12}|H_3|^2-\frac{1}{4}|F_2|^2\right)$$

where

H_3 is the NS 3-form field
F_2 is the heterotic gauge field strength

- The metric on $AdS_3 \times S^3 \times T^4$ is as follows

$$ds_{10}^2 = ds_{AdS_3}^2 + e^{2\chi} d\Omega_3^2 + ds_{T^4}^2$$

where χ is the volume modulus.

- $d\Omega_3^2$ is the metric on a sphere with radius L.

- The potential in three dimensions is as follows

$$-\frac{1}{2\kappa_3^2}\int d^3x\sqrt{-\hat{g}_3}\,V_{\text{tree}}(\phi,\chi)$$

where

$$V_{\text{tree}}(\phi,\chi)=\mathcal{V}^{-2}\left[-\frac{6}{L^2}e^{-2\chi}+\frac{2\alpha'^2 n_5^2}{L^6}e^{-6\chi}+\mathcal{V}^{-2}\frac{512\pi^8\alpha'^2 g_s^4 n_1^2}{v^2L^6}\right].$$

with

$$\mathcal{V}=e^{-2\phi+3\chi}\qquad \kappa_3^2=\frac{\kappa_{10}^2}{2\pi^2L^3v\alpha'^2}\qquad \text{vol}(T^4)=v\alpha'^2$$

where n_1 is the flux on AdS_3 and n_5 is the flux on S^3

- Set the derivatives to zero

$$\partial_\phi V_{\text{tree}} = -\frac{8e^{4(-3\chi+\phi)}}{L^6 v^2}\left(3L^4 v^2 e^{4\chi} - 512 g_s^4 n_1^2 \pi^8 \alpha'^2 e^{4\phi} - n_5^2 v^2 \alpha'^2\right) = 0$$

$$\partial_\chi V_{\text{tree}} = \frac{24e^{4(-3\chi+\phi)}}{L^6 v^2}\left(2L^4 v^2 e^{4\chi} - 256 g_s^4 n_1^2 \pi^8 \alpha'^2 e^{4\phi} - n_5^2 v^2 \alpha'^2\right) = 0$$

- Solve for $(\phi = \chi = 0)$ to get

$$L^4 = n_5^2 \alpha'^2 \qquad g_s^2 = \frac{v n_5}{(2\pi)^4 n_1} \qquad V_{\text{min}} = -\frac{2}{\alpha' |n_5|} \qquad \Lambda = \frac{1}{2} V_{\text{min}} = -\frac{1}{\alpha' |n_5|}$$

One loop correction

- The loop one correction to the potential is as follows

$$V_{1-\text{loop}} = 2\lambda \mathcal{V}^{-3} e^{3\chi} \frac{g_s^2}{\alpha'},$$

where

$$\lambda = -\frac{\pi}{2v} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} Z(\tau)$$

with $Z(\tau)$ being the partition function on a torus

- Set the derivatives to zero

$$\partial_\phi V = 0|_{\phi=0,\chi=0} = 0 \Rightarrow 2\alpha'^2\left(n_5^2 + \frac{2(2\pi)^8 g_o^4 n_1^2}{v^2}\right) - 6L_o^4 + \frac{3(2\pi)^4\lambda g_o^2 L_o^6}{\alpha' v} = 0$$

$$\partial_\chi V = 0|_{\phi=0,\chi=0} = 0 \Rightarrow 2\alpha'^2\left(n_5^2 + \frac{(2\pi)^8 g_o^4 n_1^2}{v^2}\right) - 4L_o^4 + \frac{(2\pi)^4\lambda g_o^2 L_o^6}{\alpha' v} = 0$$

- These equations imply that

$$g_o^2 = \frac{1}{|n_1|\alpha'}\sqrt{3L_o^4 - 2\alpha'^2 n_5^2}$$

- The solution for $L_\circ^2$ is

$$L_o^2 = \begin{cases} \frac{1}{3\sqrt{2}}\left[3\alpha'^2 n_5^2 + \sqrt{3(\mu_1+\mu_2)} + \sqrt{6\mu_1 - 3\mu_2 + \frac{18\sqrt{3}\, n_5^2\alpha'^6}{\sqrt{\mu_1+\mu_2}}\left(n_5^4 - \frac{60 n_1^2}{\lambda^2}\right)}\right]^{1/2} & \left(\frac{\lambda n_5^2}{n_1} \geq 60\right) \\ \frac{1}{3\sqrt{2}}\left[3\alpha'^2 n_5^2 - \sqrt{3(\mu_1+\mu_2)} + \sqrt{6\mu_1 - 3\mu_2 - \frac{18\sqrt{3}\, n_5^2\alpha'^6}{\sqrt{\mu_1+\mu_2}}\left(n_5^4 - \frac{60 n_1^2}{\lambda^2}\right)}\right]^{1/2} & \left(\frac{\lambda n_5^2}{n_1} \leq 60\right) \end{cases}$$

where

$$\mu_1 = 3\alpha'^4 n_5^4 + \frac{24 n_1^2 \alpha'^4}{\lambda^2} \qquad \mu_2 = \zeta\sqrt[3]{4} + \frac{72\sqrt[3]{2} n_1^2 \alpha'^8}{\lambda^4 \zeta}\left(n_1^2 - 6 n_5^4 \lambda^2\right)$$

$$\zeta = \frac{\alpha'^4}{\lambda^2}\left(-1458 n_5^8 n_1^2 \lambda^4 + 3888 n_5^4 n_1^4 \lambda^2 - 432 n_1^6 + 162 n_1^2 n_5^4 \lambda^2 \sqrt{81 n_5^8 \lambda^4 - 144 n_1^4 + 1104 n_1^2 n_5^4 \lambda^2}\right)^{1/3}$$

- The small n_1 behavior is concerning. We check that it is well behaved

$$L_o^2 = \sqrt{\frac{2}{3}}\alpha' n_5 \left(1 + \frac{3n_1^2}{8n_5^4\lambda^2} - \frac{261 n_1^4}{128 n_5^8 \lambda^4} + ...\right)$$

$$g_o^2 = \sqrt{\frac{3}{2}}\frac{1}{|\lambda n_5|}\left(1 - \frac{21 n_1^2}{8n_5^4\lambda^2} + \frac{2115 n_1^4}{128 n_5^8 \lambda^4} + ...\right)$$

- The minimum of the potential is as follows

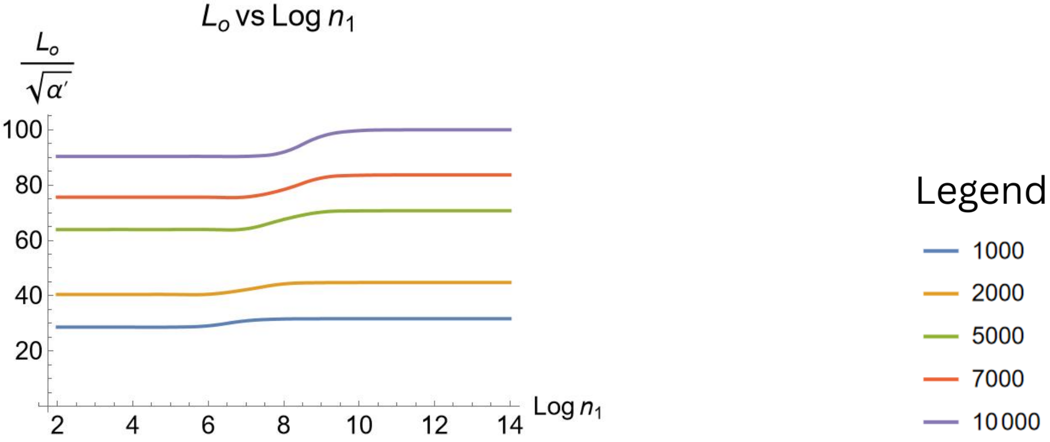
$$V_{\text{min}} = -\frac{2}{L_o^2} + \frac{g_o^2 \lambda}{\alpha'}$$

- Spoiler: The uplift to deSitter still doesn't happen

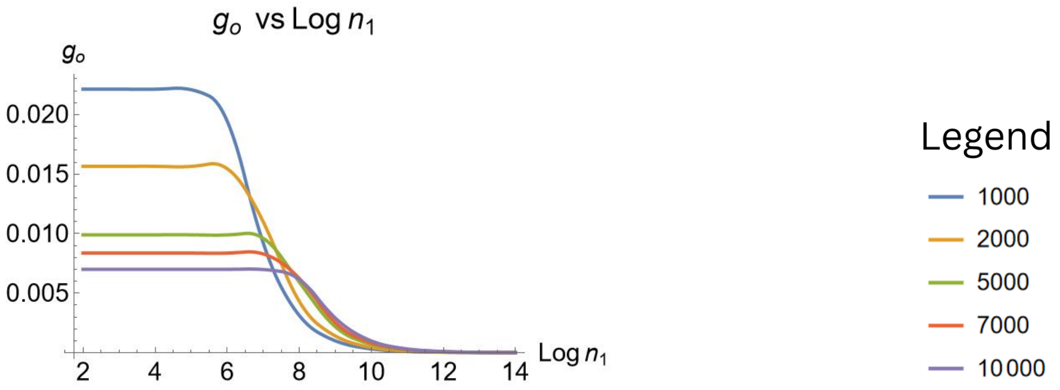
- We have the following relation

$$L_{AdS,o}^2 = -\frac{2}{V_{\text{min}}} = \left(\frac{1}{L_o^2} - \frac{\lambda g_o^2}{2\alpha'}\right)^{-1}$$

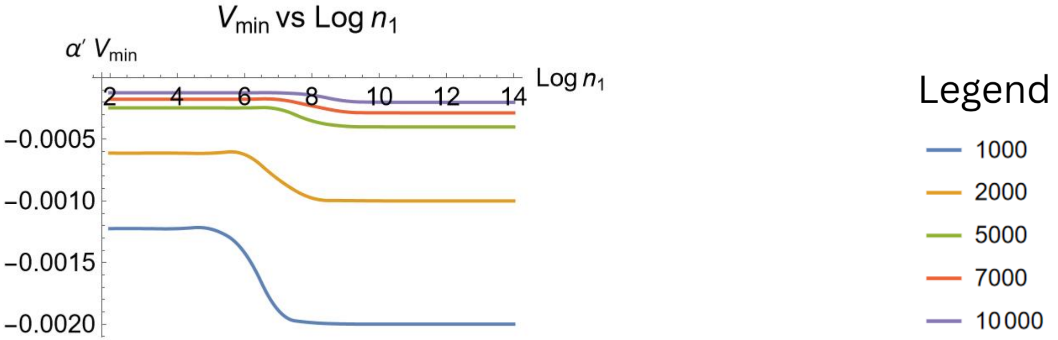
$L_{\circ}$ vs. $log(n_1)$ (different n_5 values)



$g_\circ$ vs. $log(n_1)$ (different n_5 values)



V_{min} vs. $log(n_1)$ (different n_5 values)



Background solution

- The six dimensional effective action is as follows

$$S_6 = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g} \left[e^{-2\phi} \left(R + 4 (\partial\phi)^2 - \frac{1}{12} |H_3|^2 - \frac{1}{4} |F_2|^2 - \frac{1}{2} |\partial\sigma|^2 \right) - \Lambda - \frac{1}{2} \mu_{\alpha\beta} \sigma^\alpha \sigma^\beta \right].$$

where

$$\Lambda = \frac{2\lambda g_o^2}{\alpha'}$$

- σ^α are scalars (these are 80 scalars coming from the T_4 compactification)

No-go theorem

- The argument of the no-go theorem in [Basile, Lanza; 2020] can be repeated for $M_3 \times S^3$ where M_3 is a 3D maximally symmetric space ($R_{\mu\nu} = \Omega g_{\mu\nu}$) to give

$$\int_{S^3} \Omega = -\frac{1}{36} \int_{S^3} e^{-2\phi} |H_3|^2 + \frac{1}{24} \int_{S^3} e^{-2\phi} |H_3|^2_{\mu\nu} g^{\mu\nu}$$

where $|H_3|^2_{\mu\nu} g^{\mu\nu} = H_{\mu MN} H^{\mu MN}$

- For maximally symmetric space $H_{\mu\nu\rho} = k\epsilon_{\mu\nu\rho}$ and thus,

$$\int_{S^3} \Omega = -\frac{1}{36} \int_{S^3} e^{-2\phi} \left(|H_3|^2 + 9k^2\right)$$

which rules out deSitter vacuua.

- Equations of motion (for $AdS_3 \times S^3$) give the following (with $\phi = 0, \sigma = 0, A^i_M = 0$)

$$R_{\mu\nu} = -\frac{2}{L^2_{AdS,\circ}} g_{\mu\nu} \qquad R_{\mu a} = 0 \qquad R_{ab} = \frac{2}{L^2_\circ} g_{ab}$$

$$\Rightarrow R = -\frac{6}{L^2_{AdS,\circ}} + \frac{6}{L^2_\circ}$$

and

$$H_3 = \frac{2}{\mathcal{L}_{AdS,\circ}} \epsilon_3 + \frac{2}{\mathcal{L}_\circ} \epsilon_{S^3}$$

with

$$\mathcal{L}_\circ^{-2} = L_\circ^{-2} + \frac{1}{4}\Lambda \qquad \mathcal{L}_{AdS,\circ}^{-2} = L_\circ^{-2} - \frac{1}{2}\Lambda$$

Perturbations

- Introduce perturbations as follows

$$\delta g_{\mu\nu} = H_{\mu\nu} + M g_{\mu\nu} \qquad \delta g_{\mu a} = S_{\mu a} \qquad \delta g_{ab} = K_{ab} + N g_{ab}$$

$$\delta B_{\mu\nu} = \epsilon_{\mu\nu\rho} U^{\rho} \qquad \delta B_{\mu a} = C_{\mu a} \qquad \delta B_{ab} = \epsilon_{abc} V^{c}$$

where $H_{\mu\nu}$ and K_{ab} are traceless

- Impose the Lorentz like condition $\nabla^a \delta g_{\mu a} = \nabla^b \delta g_{ab} = 0$ which translates as follows

$$\nabla^a S_{\mu a} = 0 \qquad \nabla^b K_{ab} = 0$$

- Also, we fix the B_M field gauge to get

$$\nabla^a C_{\mu a} = 0 \qquad \nabla_{[a} V_{b]} = 0$$

Spherical harmonics decomposition

- Decompose the deformations in different spherical harmonics over S_3

$$\phi = \sum_{\ell=0}^{\infty} \phi^{(\ell,0)} Y^{(\ell,0)}$$

$$S_{\mu a} = \sum_{\ell=1}^{\infty} \left(S_{\mu}^{(\ell,1)} Y_a^{(\ell,1)} + S_{\mu}^{(\ell,-1)} Y_a^{(\ell,-1)} + S_{\mu}^{(\ell,0)} \nabla_a Y^{(\ell,0)} \right)$$

$$K_{ab} = \sum_{\ell=2}^{\infty} \Big(K^{(\ell,2)} Y_{ab}^{(\ell,2)} + K^{(\ell,-2)} Y_{ab}^{(\ell,-2)} + K^{(\ell,1)} \nabla_{(a} Y_{b)}^{(\ell,1)} + K^{(\ell,-1)} \nabla_{(a} Y_{b)}^{(\ell,-1)} + K^{(\ell,0)} \nabla_{\{a} \nabla_{b\}} Y^{(\ell,0)} \Big)$$

- Imposing gauge conditions imply

$$S_{\mu}^{(\ell,0)} = 0,\; K^{(\ell,\pm1)} = K^{(\ell,0)} = 0,\; C_{\mu}^{(\ell,0)} = 0,\; V^{(\ell,\pm1)} = 0$$

- One equation of motion implies

$$\ell(\ell+2)(\ell+3)(\ell-1)\left(3\mathcal{M}+\mathcal{N}-4\Phi\right)=0$$

where

$$\mathcal{M}=M^{(\ell,0)} \qquad \mathcal{N}=N^{(\ell,0)} \qquad \Phi=\phi^{(\ell,0)}$$

- Therefore, for $\ell \geq 2$

$$3\mathcal{M}+\mathcal{N}-4\Phi=0$$

but for $\ell=0,1$ we can't impose this condition.

Scalars: $\ell \geq 2$ case

- The scalar equations can be solved to get the following mass matrix

$$\Box_0 \mathcal{M} = \left(\frac{(\ell+4)(\ell-2)}{L_o^2} + \frac{13\Lambda}{2} \right) \mathcal{M} + \left(\frac{16}{3L_o^2} + \frac{11\Lambda}{6} \right) \mathcal{N} + \frac{8\mathcal{U}}{3} - \frac{4\mathcal{V}}{3}$$

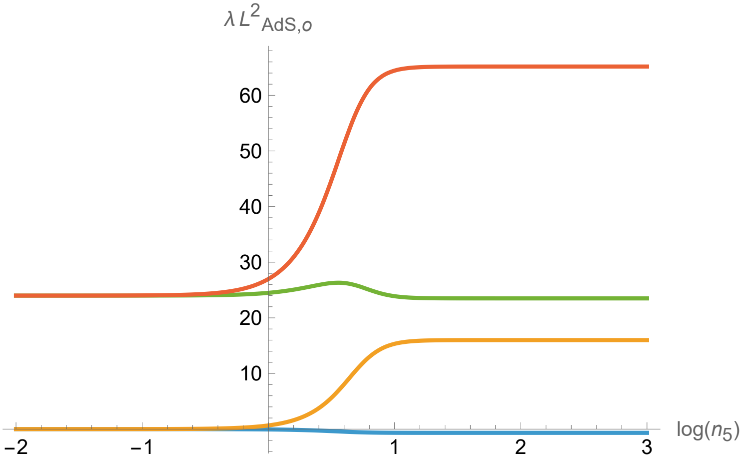
$$\Box_0 \mathcal{N} = \frac{3\Lambda}{2} \mathcal{M} + \left(\frac{8+\ell(\ell+2)}{L_o^2} + \frac{7\Lambda}{2} \right) \mathcal{N} - 4\mathcal{V}$$

$$\Box_0 \mathcal{U} = \frac{6\mathcal{M}}{\mathcal{L}^2_{AdS,o}} \left(6\Lambda + \frac{(\ell+4)(\ell-2)}{L_o^2} \right) + \frac{4}{\mathcal{L}^2_{AdS,o}} \left(\Lambda - \frac{(\ell+4)(\ell-2)}{2L_o^2} \right) \mathcal{N} + \left(\frac{\ell(\ell+2)+16}{L_o^2} - 8\Lambda \right) \mathcal{U}$$

$$\Box_0 \mathcal{V} = -\frac{4\ell(\ell+2)}{L_o^2 \mathcal{L}_o^2} \mathcal{N} + \frac{\ell(\ell+2)}{L_o^2} \mathcal{V}$$

- The eigenvalues of this matrix give the masses for small perturbations

Eigenvalues for $\ell = 2$ $(n_1 = 10)$



Scalars: $\ell = 1$ case

- For $\ell = 1$, the mass matrix is as follows

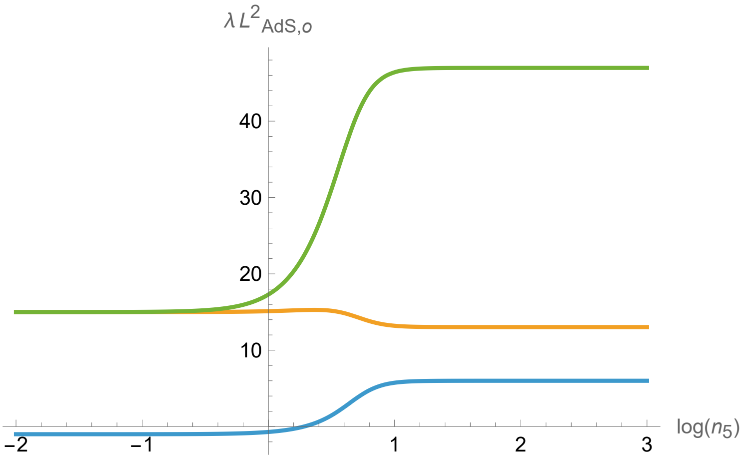
$$\Box_0\mathcal{M} = \left(\frac{5}{L_o^2} + 12\Lambda - 2L_o^2\Lambda^2\right)\mathcal{M} + \frac{8L_o^2}{3\mathcal{L}_{AdS,o}^2}\mathcal{U} - \frac{8}{3}\left(\frac{5}{L_o^2} + \Lambda + \frac{L_o^2\Lambda^2}{2}\right)\Phi$$

$$\Box_0\mathcal{U} = \left(\frac{111\Lambda}{2L_o^2} - \frac{15}{L_o^4} - 30\Lambda^2 + 3L_o^2\Lambda^3\right)\mathcal{M} + \left(\frac{19}{L_o^2} - 12\Lambda + 2L_o^2\Lambda^2\right)\mathcal{U} - 4L_o^2\mathcal{L}_{AdS,o}^{-2}(\Lambda^2 + 5L_o^{-4})\Phi$$

$$\Box_0\Phi = \left(3\Lambda - \frac{15}{2L_o^2}\right)\mathcal{M} + 2\mathcal{U} + \left(\frac{5}{L_o^2} + 2\Lambda\right)\phi$$

- The fourth eigenvalue is unphysical (i.e. it can be removed by gauge transformations)

Eigenvalues for $\ell = 1$ ($n_1 = 10$)



$\ell = 0$ case

- The mass matrix for $\ell = 0$ case is given as follows

$$\Box_0 \mathcal{N} = (8L_\circ^{-2} + 3\Lambda)\mathcal{N} + 2\Lambda\Phi$$

$$\Box_0 \Phi = \frac{9\Lambda}{2}\mathcal{N} + (8L_\circ^{-2} - \Lambda)\Phi$$

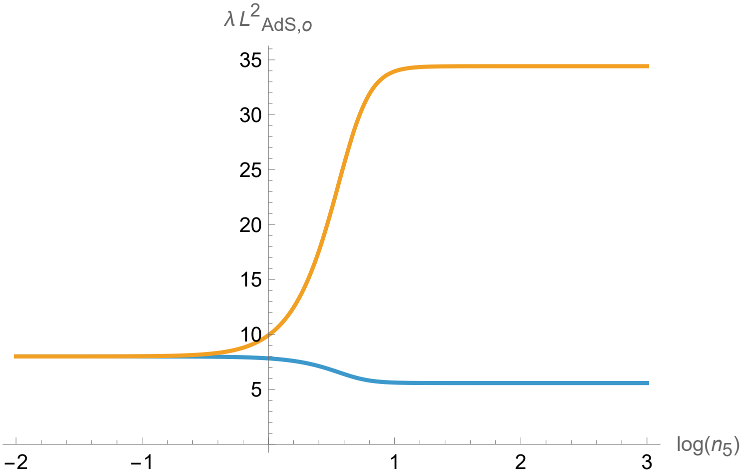
which gives the following eigenvalues

$$\lambda_{1,2} = \frac{8}{L_\circ^2} + (1 \pm \sqrt{13})\Lambda$$

- Observe that the eigenvalues are positive

$$\lambda_{1,2} > 8\left(\frac{1}{L_\circ^2} - \frac{\Lambda}{2}\right) = 8\mathcal{L}_{AdS,\circ}^{-2} > 0$$

Eigenvalues for $\ell = 0$ $(n_1 = 10)$



Small n_5 eigenvalues

- Eigenvalues depend on one dimensionless combination which is

$$s = \frac{\lambda n_5^2}{n_1}$$

- The small n_5 eigenvalues are (same as $\lambda \to 0$ limit)

$$L_{AdS}^2 \lambda_{1,2} = \ell(\ell - 2)$$

$$L_{AdS}^2 \lambda_{3,4} = (\ell + 2)(\ell + 4)$$

Large n_5 eigenvalues

- The large n_5 eigenvalues are as follows

$$\lim_{n_5\to\infty} L^2_{AdS,o}\lambda_1 = 2\ell(\ell+2)$$

$$\lim_{n_5\to\infty} L^2_{AdS,o}\lambda_2 = \frac{2}{3}\left(20+6\ell+3\ell^2-2(\xi+\sqrt{3(1-\xi^2)})\sqrt{2(32+18\ell+9\ell^2)}\right)$$

$$\lim_{n_5\to\infty} L^2_{AdS,o}\lambda_3 = \frac{2}{3}\left(20+6\ell+3\ell^2-2(\xi-\sqrt{3(1-\xi^2)})\sqrt{2(32+18\ell+9\ell^2)}\right)$$

$$\lim_{n_5\to\infty} L^2_{AdS,o}\lambda_4 = \frac{2}{3}\left(20+6\ell+3\ell^2+4\xi\sqrt{2(32+18\ell+9\ell^2)}\right)$$

where

$$\xi = \frac{1+(\sqrt{2A}\,(115+27\ell^2+54\ell)+\sqrt{B})^{2/3}}{2(\sqrt{2A}\,(115+27\ell^2+54\ell)+\sqrt{B})^{1/3}}$$

with

$$A = \frac{1}{(32+18\ell+9\ell^2)^3} \qquad B = -\frac{81(78+376\ell+500\ell^2+384\ell^3+186\ell^4+54\ell^5+9\ell^6)}{(32+18\ell+9\ell^2)^3}$$

- For $\ell = 2$ we have

$$\lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_1^{(\ell=2)} = 16 \qquad \lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_2^{(\ell=2)} = -0.669$$

$$\lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_3^{(\ell=2)} = 23.496 \qquad \lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_4^{(\ell=2)} = 65.173$$

- For $\ell = 1$, we get the following;

$$\lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_1^{(\ell=1)} = 6 \qquad \lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_3^{(\ell=1)} = 13.03$$

$$\lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_4^{(\ell=1)} = 46.97$$

- For $\ell = 0$, we get the following;

$$\lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_3^{(\ell=0)} = 5.58 \qquad \lim_{n_5 \to \infty} L^2_{AdS,o} \lambda_4^{(\ell=0)} = 34.4$$

Series expansion of eigenvalues

- A series expansion of eigenvalues can be written down (using $\omega = sv/(2\pi)^4$ as the expansion parameter), which matches the numerical results

$$L^2_{AdS,o}\lambda_{1,2} = \ell(\ell-2) + \frac{\omega\ell}{2(\ell+1)}\left(\ell^2+2\pm\sqrt{16+9\ell^2}\right)$$

$$-\frac{\omega^2\ell}{16(\ell+1)^3}\left(4\ell^4+3\ell^3-8\ell^2+8\ell+32\pm\frac{27\ell^4+36\ell^3+28\ell^2+104\ell+160}{\sqrt{16+9\ell^2}}\right)+\mathcal{O}(\lambda^3)$$

$$L^2_{AdS,o}\lambda_{3,4} = (\ell+2)(\ell+4) + \frac{\omega(\ell+2)}{2(\ell+1)}\left(\ell^2+4\ell+6\pm\sqrt{52+36\ell+9\ell^2}\right)$$

$$-\frac{\omega^2(\ell+2)}{16(\ell+1)^3}\left[4\ell^4+29\ell^3+70\ell^2+52\ell+24\pm\frac{27\ell^4+180\ell^3+460\ell^2+440\ell+208}{\sqrt{52+36\ell+9\ell^2}}\right]+\mathcal{O}(\lambda^3)$$

Vectors: $l \geq 1$

- The vector equations of motion become the following

$$\Box_0 S_\mu^{(\ell,\pm 1)} = -\frac{2}{\mathcal{L}_{AdS,\circ}}\epsilon_\mu{}^{\nu\rho}\nabla_\nu C_\rho^{(\ell,\pm 1)} + \left(\frac{(\ell^2+2\ell-1)}{L_\circ^2}+\frac{3}{2}\Lambda\right)S_\mu^{(\ell,\pm 1)} \pm \frac{2(\ell+1)\,C_\mu^{(\ell,\pm 1)}}{L_\circ \mathcal{L}_\circ},$$

$$\Box_0 C_\mu^{(\ell,\pm 1)} = -\frac{2}{\mathcal{L}_{AdS,\circ}}\epsilon_\mu{}^{\nu\rho}\nabla_\nu S_\rho^{(\ell,\pm 1)} + \left(\frac{(\ell^2+2\ell-1)}{L_\circ^2}+\frac{1}{2}\Lambda\right)C_\mu^{(\ell,\pm 1)} \pm \frac{2(\ell+1)\,S_\mu^{(\ell,\pm 1)}}{L_\circ \mathcal{L}_\circ}.$$

- Not straightforward to convert to a mass matrix
- One approach to solve the problem is in progress [Work in progress]

Stability of vector modes

- Are vector modes stable (i.e. have non-negative masses) for
  - $AdS_3 \times S^3 \times S^3$ vacuua of $O(16) \times O(16)$ heterotic theory on string scale S^1
  - $AdS_3 \times S^3$ vacuua of $O(16) \times O(16)$ heterotic theory on string scale T^4?

Torus moduli

- The one-loop correction to the potential (at an extremum σ_*) is

$$\Lambda = \frac{2\lambda g_\circ^2}{\alpha'} \text{ with } \lambda = -\frac{\pi}{2v}\int \frac{d^2\tau}{\tau_2^2} Z(\sigma_*)$$

- This gives rise to terms quadratic in σ

$$\Lambda(\sigma_* + \sigma) = \Lambda(\sigma_*) + \frac{1}{2}\left.\frac{\partial^2 \Lambda}{\partial\sigma^\alpha \partial\sigma^\beta}\right|_{\sigma_*} \sigma^\alpha \sigma^\beta + \ldots$$

Torus moduli

- If the torus moduli part of the action is

$$S \supset \frac{1}{2\kappa_6^2}\int d^6x\sqrt{-g}\left(-\frac{1}{2}e^{-2\phi}(\partial\sigma)^2 - \frac{1}{2}\mu_{\alpha\beta}\sigma^\alpha\sigma^\beta\right)$$

$\Rightarrow$ the BF bound is $L^2_{AdS,\circ}m^2_{\sigma_*} \geq -1$ where $m^2_{\sigma_*}$ is lowest eigenvalue of $\mu_{\alpha\beta}$

- The kinetic term may not be properly normalized (e.g. for (G, B, A) moduli)

$$S \supset \frac{1}{2\kappa_6^2}\int d^6x\sqrt{-g}\left(-\frac{1}{2}e^{-2\phi}\gamma_{\alpha\beta}\partial_\mu\tilde{\sigma}^\alpha\partial^\mu\tilde{\sigma}^\beta - \frac{1}{2}\tilde{\mu}_{\alpha\beta}\tilde{\sigma}^\alpha\tilde{\sigma}^\beta\right)$$

Torus moduli

- Define $J^{\alpha}{}_{\beta}$ as $\tilde{\sigma}^{\alpha} = J^{\alpha}{}_{\beta}\sigma^{\beta}$ and then it can shown that

$$J^T \gamma J = \mathbb{I} \qquad \mu = J^T \tilde{\mu} J$$

- Using Cholesky decomposition

$$\gamma = L^T L \Rightarrow J = L^{-1} \Rightarrow \mu = (L^{-1})^T \tilde{\mu} L^{-1}$$

- Preliminary work shows BF bound violations at several extrema in the T^4 moduli space [Work in progress]

$AdS_3 \times S^3 \times K3$

- Is $AdS_3 \times S^3 \times K3$ perturbatively stable?
- Metric on $K3$ not known in closed form (numerical metrics are available)
- Hope at certain points in the $K3$ moduli space

Non-perturbative stability

- Are these $AdS_3 \times S^3 \times S^3$ and $AdS_3 \times S^3$ vacuua stable non-perturbatively?
- Multiple fluxes $\rightarrow$ probably giant leaps in bubbles [Bousso & Polchinski; 2000][Brown & Dahlen; 2022]
- Can't use thin wall approximation [Coleman; 1977]
- Have to worry about torus and circle moduli space

No torus/circle moduli (Work in progress)

- We can study the stability of the $O(16) \times O(16)$ theory on $AdS_4 \times S^3 \times S^3$ with H_3 fluxes on the two S^3s $(n_5, \hat{n}_5)$ [Work in progress with Basile and Robbins]
  - No circle/torus moduli
  - We found perturbative instabilities (modes below the BF bound)
  - For non-perturbative decays, we do find vacuua with small decay rate when $n_5 = \hat{n}_5$
  - Would the presence of torus/circle moduli change this conclusion?

Thanks for listening
Questions?

Equation of motion (Dilaton)

$$\frac{1}{4}\Box_0\left(2M+3N-4\phi\right)+\frac{1}{4}\Box_x\left(3M+2N-4\phi\right)-\frac{3\Lambda}{8}(M+N)-\frac{1}{4}\nabla_\mu\nabla_\nu H^{\mu\nu}$$
$$+\frac{1}{2\mathcal{L}_o}\nabla_a V^a-\frac{1}{2\mathcal{L}_{AdS,o}}\nabla_\mu U^\mu=0$$

Equation of motion ($g_{\mu\nu}$ components)

$g_{\mu\nu}$ trace component

$$\frac{12M}{\mathcal{L}^2_{AdS,o}} + \frac{3\Lambda}{4}(M+3N-4\phi) - \frac{3}{\mathcal{L}_{AdS,o}}\nabla_\mu U^\mu - \Box_0(M+3N-4\phi)$$

$$+\frac{1}{2}\nabla_\mu\nabla_\nu H^{\mu\nu} - \frac{3}{\mathcal{L}_o}\nabla_a V^a - 3\Box_x(M+N-2\phi) = 0$$

$g_{\mu\nu}$ traceless component

$$\frac{4}{L^2_{AdS,o}} H_{\mu\nu} - \nabla_\mu\nabla_\nu(M+3N-4\phi) + 2\nabla_\rho\nabla_{(\mu}H_{\nu)}{}^\rho - (\Box_0+\Box_x)H_{\mu\nu}$$

$$+\frac{1}{3}g_{\mu\nu}\Box_0(M+3N-4\phi) - \frac{2}{3}g_{\mu\nu}\nabla_\sigma\nabla_\rho H^{\sigma\rho} = 0$$

Equation of motion (g_{ab} components)

g_{ab} trace component

$$-4L_o^{-2}N + \frac{\Lambda}{4}\left(3M - 3N - 4\phi\right) + \mathcal{L}_{AdS,o}^{-1}\nabla_\mu U^\mu - \Box_0\left(M + N - 2\phi\right) + \frac{1}{2}\nabla_\mu\nabla_\nu H^{\mu\nu}$$

$$+\mathcal{L}_o^{-1}\nabla_a V^a - \frac{1}{3}\Box_x\left(3M + N - 4\phi\right) = 0$$

g_{ab} traceless component

$$2\nabla_\mu\nabla_{(a}S^\mu{}_{b)} - \left(\Box_0 + \Box_x - 2L_o^{-2}\right)K_{ab} - \nabla_{\{a}\nabla_{b\}}\left(3M + N - 4\phi\right) = 0$$

Equation of motion ($g_{\mu a}$ component)

$$\frac{3\Lambda}{2} S_{\mu a} - (\Box_0 + \Box_x) S_{\mu a} + \nabla_\nu \nabla_a H_\mu{}^\nu - 2\mathcal{L}_o^{-1} \nabla_\mu V_a + \nabla_\mu \nabla_\nu S^\nu{}_a - 2\nabla_\mu \nabla_a (M + N - 2\phi)$$

$$+ 2\mathcal{L}_{AdS,o}^{-1} (\epsilon_{\mu\nu\rho} \nabla^\rho C^\nu{}_a + \nabla_a U_\mu) - 2\mathcal{L}_o^{-1} \epsilon_{abc} \nabla^c C_\mu{}^b = 0$$

Equation of motion (B field)

- $B_{\mu\nu}$ component

$$\mathcal{L}_{AdS,o}^{-1}\nabla^{\mu}\left(3M-3N+4\phi\right)-\nabla^{\mu}\nabla_{\nu}U^{\nu}-\Box_{x}U^{\mu}=0$$

- B_{ab} component

$$2\mathcal{L}_{o}^{-1}\nabla_{\mu}S^{\mu c}-\Box_{0}V^{c}-\epsilon^{c}{}_{ab}\nabla_{\mu}\nabla^{b}C^{\mu a}-\mathcal{L}_{o}^{-1}\nabla^{c}\left(3M-3N-4\phi\right)-\nabla^{c}\nabla_{a}V^{a}=0$$

- $B_{\mu a}$ component

$$\left(\frac{\Lambda}{2}-\Box_{0}-\Box_{x}\right)C_{\mu a}+\nabla_{\mu}\nabla_{\nu}C^{\nu}{}_{a}+2\mathcal{L}_{AdS,o}^{-1}\epsilon_{\mu\lambda\nu}\nabla^{\nu}S^{\lambda}{}_{a}-\epsilon_{\mu\lambda\nu}\nabla^{\nu}\nabla_{a}U^{\lambda}$$