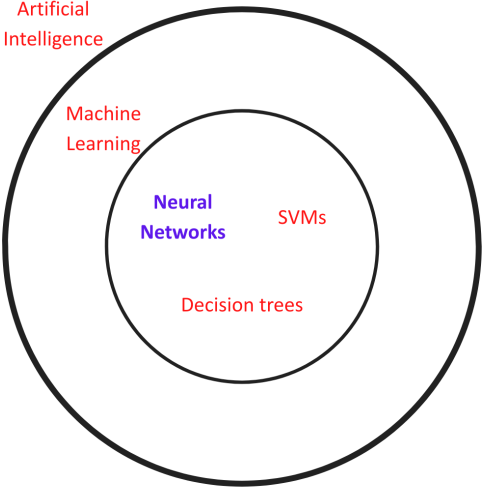
How neural networks work

Hassaan Saleem

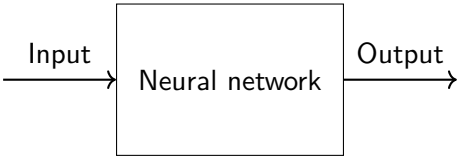
Science Ki Duniya

The AI landscape



The main aim

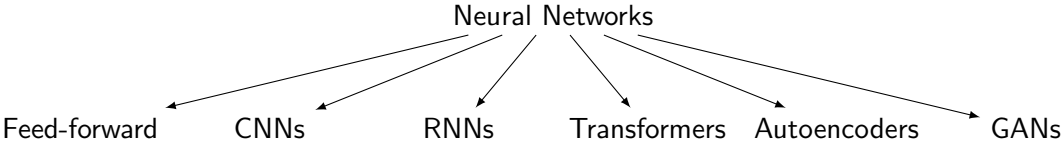
- The main aim of a neural network is to predict the output of an input



- Neural network is a **function**

$$f(input) = output$$

- How to get the right function to get a desired output?


Neural Networks
Feed-forward
CNNs
RNNs
Transformers
Autoencoders
GANs

Parameters

- How to find the **right** function?

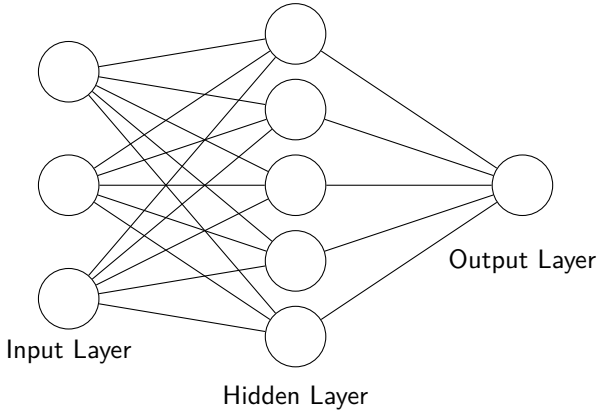
$$f[\underbrace{\ell_1, ...}_{\text{parameters}}](input) = \underbrace{output}_{\text{label}}$$

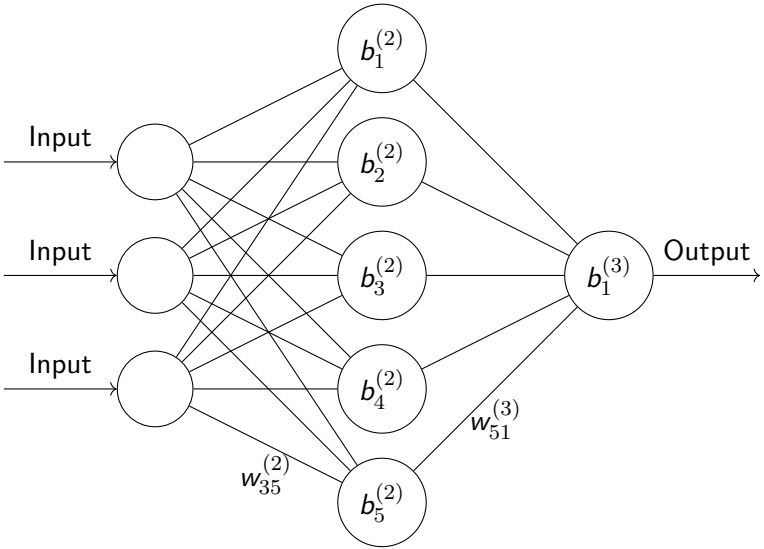
- Divide the parameters into three categories

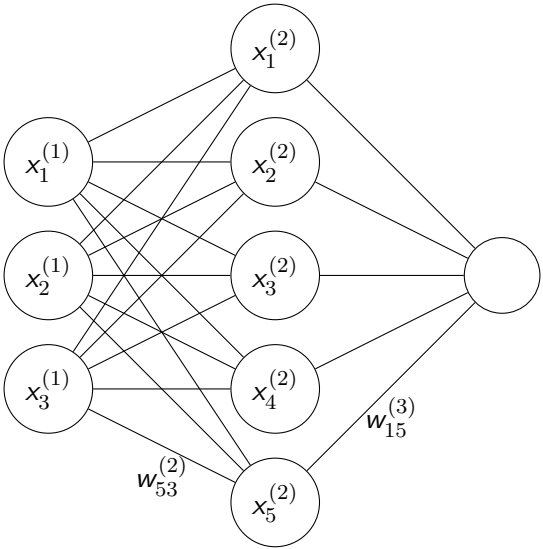
$$f[w_1, ...; b_1, ...; h_1, ...](input) = output$$

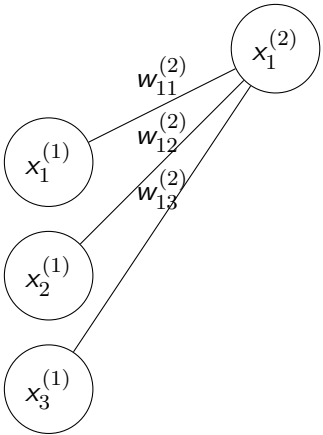
$$w_{1,...} = \text{Weights}$$
$$b_{1,...} = \text{Biases}$$
$$h_{1,...} = \text{Hyperparameters}$$

A typical feed-forward neural network

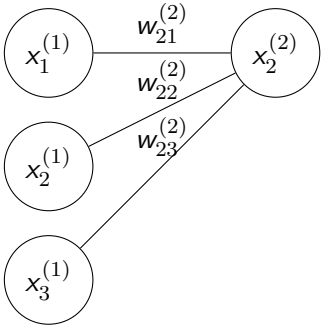



Input
Input
Input
$b_1^{(2)}$
$b_2^{(2)}$
$b_3^{(2)}$
$b_4^{(2)}$
$b_5^{(2)}$
$b_1^{(3)}$
Output
$w_{35}^{(2)}$
$w_{51}^{(3)}$

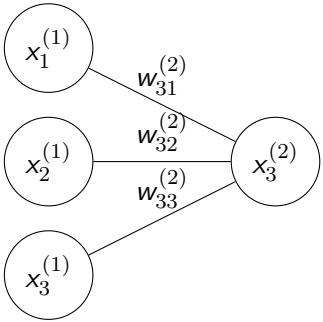

$x_1^{(2)}$
$x_1^{(1)}$
$x_2^{(2)}$
$x_2^{(1)}$
$x_3^{(2)}$
$x_3^{(1)}$
$x_4^{(2)}$
$x_5^{(2)}$
$w_{15}^{(3)}$
$w_{53}^{(2)}$



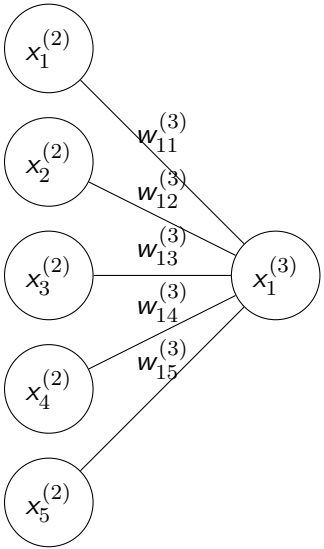
$$
\begin{aligned}
x_1^{(2)} &= f\left(w_{11}^{(2)} x_1^{(1)} + w_{12}^{(2)} x_2^{(1)} + w_{13}^{(2)} x_3^{(1)} + b_1^{(2)}\right) \\
&= f\left(\sum_{j=1}^{3} w_{1j}^{(2)} x_j^{(1)} + b_1^{(2)}\right)
\end{aligned}
$$



$$x_2^{(2)} = f\left(w_{21}^{(2)} x_1^{(1)} + w_{22}^{(2)} x_2^{(1)} + w_{23}^{(2)} x_3^{(1)} + b_2^{(2)}\right)$$
$$= f\left(\sum_{j=1}^{3} w_{2j}^{(1)} x_j^{(1)} + b_2^{(2)}\right)$$



$$x_3^{(2)} = f\left(w_{31}^{(2)} x_1^{(1)} + w_{32}^{(2)} x_2^{(1)} + w_{33}^{(2)} x_3^{(1)} + b_3^{(2)}\right)$$
$$= f\left(\sum_{j=1}^{3} w_{3j}^{(2)} x_j^{(1)} + b_3^{(2)}\right)$$



$$
\begin{aligned}
x_1^{(3)} &= f\left(w_{11}^{(3)} x_1^{(2)} + w_{12}^{(3)} x_2^{(2)} + w_{13}^{(3)} x_3^{(2)} \right. \\
&\qquad \left. + w_{14}^{(3)} x_4^{(2)} + w_{15}^{(3)} x_5^{(2)} + b_1^{(3)}\right) \\
&= f\left(\sum_{j=1}^{5} w_{1j}^{(3)} x_j^{(2)} + b_1^{(3)}\right)
\end{aligned}
$$

$$\mathbf{x}^{(i+1)} = f\left(\mathbf{w}^{(i+1)}.\mathbf{x}^{(i)} + \mathbf{b}^{(i+1)}\right)$$

$\mathbf{x}^{(i)}$ = state vector the *i*-th layer (activation vector)

$\mathbf{b}^{(i)}$ = bias vector the *i*-th layer

f = Activation function

$\mathbf{w}^{(i+1)}$ = weight matrix for the *(i+1)*-th layer

$w_{mn}^{(i+1)}$ = from *n*-th neuron in *i*-th layer to *m*-th neuron in $(i+1)$-th layer

Loss function

- The output in our example is $x_1^{(3)} = y$
- In **supervised learning**, inputs come with **labels** ℓ (correct answers)
- The goal is to minimize the difference between y and ℓ

Loss function

- You have many training examples ($p = 1, ..., P$). The output and label for p-th training example is y_p and ℓ_p, respectively
- We define the **loss function** as

$$L = \frac{1}{P} \sum_{p=1}^{P} (y_p - \ell_p)^2$$

- A neural network has to minimize this loss function

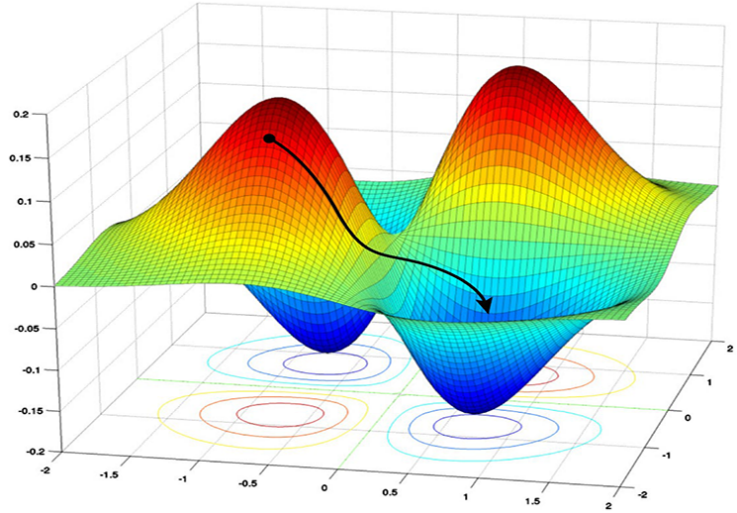
Optimization

- Begin with random weights and biases
- Update the weights and biases to reduce the loss function
- Continue until the loss function is minimum
- This procedure is called **optimization** (many algorithms are available)

- Suppose you have a function $f(x_1, ..., x_n)$ that you want to minimize (you know that there is only one minimum, and no other extrema)
- You take the derivatives with respect to all x_n and set them to zero

$$\frac{\partial f(x_1, ..., x_n)}{\partial x_j} = 0 \text{ for } j = 1, ..., N$$

- Solve these equations to get the values of $x_1, ..., x_N$ at which $f(x_1, ..., x_N)$ has a minimum


0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2
1
0
-1
-2

- We need to find weights and biases that minimize the loss function. So we have

$$\frac{\partial L}{\partial w_{mn}^{(i)}} = 0 \qquad \frac{\partial L}{\partial b_{m}^{(i)}} = 0$$

- The only thing that depends on weights and biases in L is y_p. We need to differentiate y_p.

- In our example, the output y is

$$y = f\left(\mathbf{w}^{(3)}.\mathbf{x}^{(2)} + b^{(3)}\right)$$

- We also have

$$\mathbf{x}^{(2)} = f\left(\mathbf{w}^{(2)}.\mathbf{x}^{(1)} + \mathbf{b}^{(2)}\right)$$

- So we have

$$y = f\left(\mathbf{w}^{(3)}.f\left(\mathbf{w}^{(2)}.\mathbf{x}^{(1)} + \mathbf{b}^{(2)}\right) + b^{(3)}\right)$$

- The easiest derivatives to take are the derivatives with respect to $b^{(3)}$ and $\mathbf{w}^{(3)}$, then $\mathbf{b}^{(2)}$ and $\mathbf{w}^{(2)}$

- This is called **backpropagation** (Geoffrey Hinton got a Nobel Prize for it)

- The weights and biases are changed by **weight update rule**

$$w_{mn}^{(i)}(\text{new}) = w_{mn}^{(i)}(\text{old}) - \alpha \frac{\partial L}{\partial w_{mn}^{(i)}}$$

$$b_{m}^{(i)}(\text{new}) = b_{m}^{(i)}(\text{old}) - \alpha \frac{\partial L}{\partial b_{m}^{(i)}}$$

- α is called the **learning rate** and this method update weights/biases is called **gradient descent**.

- Gradient descent isn't very desirable for many applications. Other methods exist e.g. Stochastic gradient descent (SGD), Adam, and Momentum.

Hyperparameters

- We are optimizing weights and biases only.
- We aren't optimizing
  - the number of layers
  - the learning rate
  - the number of neurons in each layer
- These parameters are called **hyperparameters**. There is no known theory to optimize these hyperparameters.

After training

- After optimization, the neural network is **trained**
- We can test is using some **test data** (in contrast to **training data**)
- If it passes the test, you can use this neural network to perform tasks you trained it for

Miscelleneous points

- There are many issues that I am not mentioning here (underfitting, overfitting, hallucination, vanishing gradient problem)
- Other kinds of neural networks (**Architectures**) are used for other purposes
  - CNNs (for context, e.g., images)
  - RNNs (for tasks that require memory, e.g. language processing)
  - Transformers (RNNs, but better, used to make ChatGPT)
- You can use **Pytorch** library in Python to make and train neural networks